7 Motion in a Circle

Introduction

Circular motion occurs in many processes, such as the motion of electrons in orbits around the atom, planets moving around the sun (approximately circular trajectory). A person in a rollercoaster doing circular motion will not fall out of the seat even when upside down. You could have certainly rotated a stone attached to a string in circular motion. You should have also noticed that if the string breaks, the stone will fly off along a tangent to the circle. The same idea you can apply to planets moving in orbits around the sun or any other bodies in the universe.

In this chapter, you will study how objects are kept moving in circular orbits.

At the end of this topic you will be able to:

(i) Demonstrate understanding of angular displacement and angular velocity,

(ii) Show the relationship between linear velocity and angular velocity,

(iii) Explain, using vector method, how centripetal force acts towards the centre of a circle,

(iv) Solve problems about bodies moving in circles.

Angular displacement, θ

Angular displacement refers to the angle moved by an object doing circular motion.
The unit of angular displacement is radians (rad).

Angular velocity, ω

Angular velocity refers to the rate of change of angular displacement.

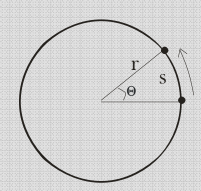
$$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

Therefore,

$$\omega = \frac{\theta\ (\text{rad})}{t\ (\text{s})}$$

Unit of angular is rads^{-1}.

In mathematics, you have learnt about circles and the relationship between r, θ and s (arc length).



$s = r\theta$, where θ is measured in radians.
The angular displacement of a particle having done half cycle. is π radians.

$\omega = \frac{d\theta}{dt}$ for non uniform change.

$\omega = \frac{\Delta\theta}{\Delta t}$ for small changes of angular displacement during small interval of time.

Linear velocity and Angular velocity

An object is made to rotate in a circle with constant linear speed v. At any instant of time, its angular velocity ω is the rate of change its angular displacement.

Using the fact that, $s = r\,\theta$

For an object doing circular motion, the arc length s=r θ, where θ is measured in radians.

If follows that

$\frac{s}{t} = r\frac{\theta}{t}$ (Including the time dimension, by dividing on both side of the equation)

Therefore, $v = r\,\omega$

EXAMPLE 1

Compare the angular velocities and linear velocities of the Earth which rotates on its axis every 24 hours and a fan which rotates 25 times in a minute. Comment on your answer.
[Assume that the radius of the fan is 30 cm]

Answer

Earth

$$\omega = \frac{\theta}{t} = \frac{2\pi}{24x3600} = 7.27x10^{-5}\ rads^{-1}$$

$$v = r\,\omega$$
$$= 6.37 \times 10^{6} \times 7.27 \times 10^{-5}$$
$$= 4.63 \times 10^{2}\ ms^{-1}$$

Can you imagine that the Earth or an object on the Earth is rotating with a linear speed of 463 m/s?

Fan

Angular velocity ω of the fan = 25x2π = 50π $radmin^{-1}$

or

$$\omega = 50\pi\frac{rad}{min} x \frac{min}{60\ s} = 2.62\ rads^{-1}$$

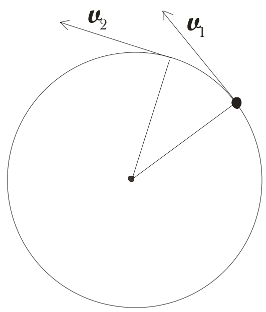
$$v = r\,\omega$$
$$= 0.30 \times 2.62$$
$$= 0.786\ ms^{-1}$$

Comment: Despite the large angular velocity, the tip of the blade of the fan has a much smaller linear velocity that the Earth because of the large difference in their radii.

Centripetal Acceleration

Consider an object is moving in a circle of radius *r* with constant angular velocity ω. The tangential speed is constant but the object's tangential velocity vector is changing since the direction is changing.

Though $|\vec{v}_1| = |\vec{v}_2|$, they have different directions and therefore their velocities are different.



In which direction does the centripetal acceleration acts?

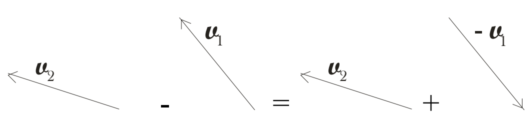
From definition, acceleration, $\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$. The acceleration should be acting in the same direction as $\Delta\vec{v}$. It is therefore required to show the direction of $\Delta\vec{v}$; acceleration a will also acts in the same direction as $\Delta\vec{v}$.

Note that in vector notation, $\vec{a} = k\vec{b}$ implies that vector *a* is in the same direction a vector *b*.

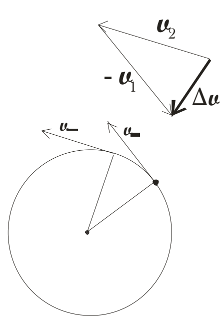
$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

$$\Delta\vec{v} = \vec{v}_2 + (-\vec{v}_1)$$

It is more convenient to add vectors rather than subtract them. Pay attention to the vector, they have been displaced parallel to the velocity vec



Adding the two vectors, it follows that:



Pay attention to the direction of the change in velocity. It acts towards the centre.

From the vector addition, it follows that the acceleration when the object is doing circular motion acts towards the center of the circle. This acceleration is known as the **centripetal acceleration** and it always act towards the center of a circle.

Verify for whatever velocity vectors we are considering, the acceleration always act towards the centre. You need to be careful in selecting the velocity vectors which should be very close to each other since Δt is very small.

Centripetal acceleration is defined as the rate of change of tangential velocity.

$$\begin{aligned} a &= \frac{v}{t} = \frac{r\omega}{t} = \frac{r}{t}\omega \\ &= v\omega = (r\omega)\omega \\ &= r\omega^2 \end{aligned}$$

Centripetal force should not be confused with centrifugal force. The centrifugal force is a fictitious force that arises from being in a rotating reference frame. It exists, or rather seems to exist, only in a non-inertial coordinate system (one which is accelerating or rotating) where Newton's laws are not valid. 'Centrifugal force' is simply a manifestation of **inertia**.

Alternately,

$$a = r\omega^2 = r(\frac{v}{r})^2 = \frac{v^2}{r}$$

Centripetal force

Centripetal force means 'centre seeking' force. In the absence of that force, any object will be moving in a straight line.

Following Newton's second law, it follows that the presence of this acceleration is due to a net force. This force which acts towards the center of the circle is called the **centripetal force**. $F = mr\omega^2$

If you are sitting in a car which is on the point of making a round about to the left. The car then makes a circular trajectory. The frictional force at the wheels gives rise to an unbalanced force on the car thus producing an acceleration which is directed towards the centre of the circular trajectory. Since you were in motion, your body tends to stay in motion. This tendency to resist the acceleration causes your body to continue in its straight line motion. So, your body will lean to the right and be prevented to do so by the outside (right) door (in case you were seated on the back right seat). You may think that you are being accelerated away from the circle.

Diagram of car making a round-about to the left showing a person seated on the back right seat.

Inertia is responsible for keeping your body in motion.

You are in fact undergoing your straight line path, tangential to the circle while the car is accelerating away from you, that is, towards the centre of the circle.

Centripetal force is not a new type of force but is rather a net force.

This is a false sensation that you may have that there is a force pushing you off the centre of the circle.
In this case there is no physical object that is capable of pushing you outwards, that is away from the centr of the corcle.

EXAMPLE 2

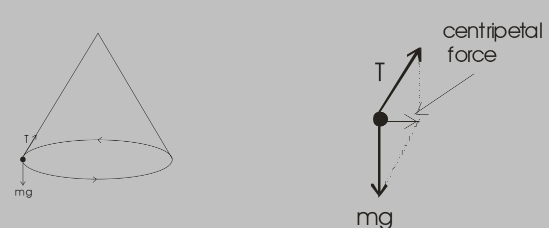
Explain the occurrence of centripetal force in the following situations:

1. A car making a turn
2. a stone undergoing circular motion in a horizontal direction
3. a bucket of water which is tied to a string is spun in a vertical direction

Answer

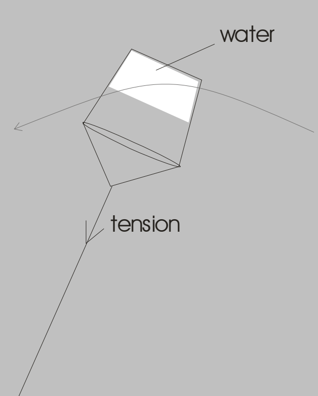
1.
Diagram of the first situation.
When the car is making the turn, the frictional force at the wheels provides the centripetal force required for circular motion.

2.



When the stone is undergoing circular motion, weight and tension contribute to the occurrence of the centripetal force. Here again, centripetal force is not a new type of force, but a net force.

3.



Both the water and bucket continue to move in a straight line following Newton's first law of motion but the angular velocity is such that the outward force on the bucket due to

Note that when the bucket is set in circular motion, the water has a tendency to move off in a straight line away from the centre according to Newton's first law but is prevented to do so by the bucket which pull it towards the centre.

the water inside is greater than the gravitational attraction the bulk of the water experiences when the bucket is upside down at the maximum height.

At the top of the circle, the centripetal force is supplied by the tension and the weight of the water.

$$T + mg = m\frac{v^2}{r}$$

Similarly, at the bottom of the circle,

$$T - mg = m\frac{v^2}{r}$$

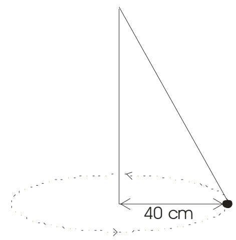
It the speed is not sufficiently hight, the water will spurt out of the bucket once it is at the top of the circle.

The string holding the bucket is more likely to break when the bucket is at the bottom since tension is maximum.

End of Chapter Questions

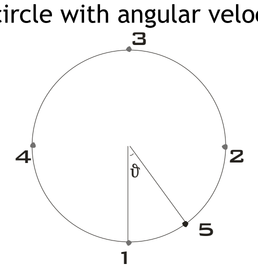
1. A stone, of mass m, is attached to a string and whirled in a vertical circle of radius r. When the stone is at the bottom of the circle, the tension in the string is 3 times the stone's weight. What is the speed of the stone. Give your answer in terms of g and r.

2. A body of mass 1 kg is attached to a string 1m long and moves in a horizontal circle of radius 40 cm.



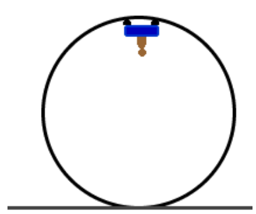
(a) Draw the forces acting on the mass.
(b) Explain the occurrence of the centripetal force.
(c) Calculate the magnitude of the tension in the string.
(d) Calculate the period of the mass.

3. A string of 1 m long is used to whirled a stone of mass 100 g in a vertical circle with angular velocity of 4.0 rads^{-1}.



(a) Draw the forces acting on the mass at position 1, 2 ,3 & 4.
(b) Determine the tensions in the string at position 1, 2, 3 & 4.
(c) At which position the string is most likely to break? Explain.
(d) Draw the force acting on the mass at position 5.
(e) Write down a formula the tension when the stone is at position 5.
(f) Uisng the formula from (e), confirm your answer to (b).

4. A man of mass 70 kg in a roller coaster car is stting in a seat on a bathroom scale. His speed at the top point of the circle is 40 ms^{-1} and the radius of the coaster track is 90 m.



(a) Explain why the man does not fall off the seat.
(b) Determine the magnitude of the force shown on the scale.